4.2 Pg. 167 #1, 3, 5, 7, 9, 11, 13, 17, 19

1. $x^2y + xy^2 = 6$

$$2xy + x^2\frac{dy}{dx} + y^2 + x \cdot 2y\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

3. $y^2 = \dfrac{x-1}{x+1}$

$$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{x+1-x+1}{2y(x+1)^2}$$

$$\frac{dy}{dx} = \frac{2}{2y(x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$$

5. $x = \tan y$

$$1 = \sec^2 y \cdot \frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \cos^2 y$$

7. $x + \tan(xy) = 0$

$$1 + \sec^2(xy)\left(y + x\frac{dy}{dx}\right) = 0$$

$$1 + y\sec^2(xy) + x\sec^2(xy)\frac{dy}{dx} = 0$$

$$x\sec^2(xy)\frac{dy}{dx} = -1 - y\sec^2(xy)$$

$$\frac{dy}{dx} = \frac{-1 - y\sec^2(xy)}{x\sec^2(xy)}$$

9. $x^2 + y^2 = 13 \qquad (-2, 3)$

$$2x + 2y\frac{dy}{dx} = 0 \qquad x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x \qquad \frac{dy}{dx} = \frac{-x}{y}$$

at $(-2, 3)$ $\quad \frac{dy}{dx} = \frac{2}{3}$

11. $(x-1)^2 + (y-1)^2 = 13 \qquad (3,4)$

$$2(x-1) + 2(y-1)\frac{dy}{dx} = 0$$

$$(x-1) + (y-1)\frac{dy}{dx} = 0$$

$$(y-1)\frac{dy}{dx} = 1-x$$

$$\frac{dy}{dx} = \frac{1-x}{y-1}$$

at $(3,4)$ $\quad \frac{dy}{dx} = \frac{1-3}{4-1} = \frac{-2}{3}$

13. $x^2y - xy^2 = 4$

$$2xy + x^2\frac{dy}{dx} - y^2 - x\cdot 2y\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 2xy$$

$$\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$$

$\frac{dy}{dx}$ is undefined at $x^2-2xy = 0$

$x(x-2y) = 0$ therefore at $x=0$ and $x-2y=0$

$x = 2y \qquad (2y)^2y - 2y\cdot y^2 = 4 \qquad 4y^3 - 2y^3 = 4$

$2y^3 = 4 \qquad y^3 = 2 \qquad y = \sqrt[3]{2}$ therefore $x = 2\sqrt[3]{2}$

$\frac{dy}{dx}$ is defined everywhere except at $x=0$ and at $\left(2\sqrt[3]{2}, \sqrt[3]{2}\right)$

17. $x^2 + xy - y^2 = 1 \quad (2,3)$

$$2x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x-2y}$$

at $(2,3)$ $\frac{dy}{dx} = \frac{-2(2)-3}{2-2(3)} = \frac{-7}{-4} = \frac{7}{4}$

Tangent line. $y - 3 = \frac{7}{4}(x-2)$

Normal line $y - 3 = -\frac{4}{7}(x-2)$

19. $x^2y^2 = 9 \quad (-1,3)$

$$2xy^2 + x^2 \cdot 2y\frac{dy}{dx} = 0$$

$$2x^2y \cdot \frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = \frac{y}{x}$$

at $(-1,3)$ $\frac{dy}{dx} = \frac{3}{-1} = -3$

Tangent line $y - 3 = -3(x+1)$

Normal line $y - 3 = \frac{1}{3}(x+1)$